Neumann to Steklov eigenvalues: an asymptotic analysis.

We consider a spectral problem for the Laplace operator with Neumann boundary conditions in a smooth domain Ω of $\mathbb{R}^2$. The equation involves a parameter ρ_ε which is a positive measurable function and plays the role of a mass density. This *density coefficient* is piecewise constant and is of order ε^{-1} in a ε-neighborhood of the boundary, as ε goes to zero, while it is of order ε in the rest of Ω. Moreover, its integral over the whole of Ω is fixed and does not depend on ε. We provide asymptotics of the eigenvalues and the eigenfunctions as $\varepsilon \to 0$ and find explicit formulas for the zero and first order terms in the expansion, which are solutions of suitable auxiliary problems. In particular, it turns out that the Neumann eigenvalues converge to the appropriate Steklov eigenvalues as $\varepsilon \to 0$. Note that the convergence result for the eigenvalues can also be proved with other techniques, see e.g., [1, 2]. Moreover, we obtain additional informations on the rate of convergence of the eigenvalues which is of order ε and we provide an explicit formula for the so-called *topological derivative* of the Neumann eigenvalues at $\varepsilon = 0$ (see [3]). Finally, in the case that $\Omega = B$ is the unit ball we prove that the derivative of the eigenvalues at $\varepsilon = 0$ is positive and then conclude that *"the Steklov eigenvalues locally minimize the Neumann eigenvalues"* for ε small enough. This result can be also proved using the explicit characterization of the Neumann eigenfunctions of the unit ball in terms of Bessel functions and the Implicit Function Theorem (see [4, 5]).
All the results contained here can be found in [2, 3, 4, 5].

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