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On the spectral behavior of the Neumann Laplacian under mass density perturbation

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Abstract

Let Ω be a bounded open set in $\mathbb{R}^N$ of class C^2. We consider the classical eigenvalue problem

$$\begin{cases} -\Delta u = \lambda \rho u, & \text{in } \Omega, \\ \frac{\partial u}{\partial \nu} = 0, & \text{on } \partial\Omega, \end{cases}$$

in the unknowns u (the eigenfunction) and λ (the eigenvalue). The parameter ρ is a positive bounded measurable function which can be understood as a mass density, in which case $M = \int_\Omega \rho dx$ represents the corresponding total mass. We discuss stability results for the dependence of the eigenvalues and eigenfunctions upon variation of ρ. In particular, we consider the case where M is fixed and $\rho = \rho_\varepsilon$, $\varepsilon > 0$ is a family of mass densities concentrating near the boundary of Ω as $\varepsilon \to 0$. We prove norm resolvent convergence of such Neumann problems to the appropriate limiting Steklov problem. In the case where Ω is a ball, explicit computations allow to prove differentiability of the eigenvalues upon variation of ε and provide formulas for the derivatives at $\varepsilon = 0$.

Bibliography

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