

Luigi Provenzano Dip. Matematica, Università degli Studi di Padova, via Trieste 63, 35126 Padova, Italy, email: proz@math.unipd.it

On the fundamental tones of free vibrating plates

Abstract

Let Ω be a bounded domain in \mathbb{R}^N of class C^1 and $\tau > 0$ be fixed. We consider the following Steklov-type eigenvalue problem for the biharmonic operator

$$(1) \quad \begin{cases} \Delta^2 u - \tau \Delta u = 0, & \text{in } \Omega, \\ \frac{\partial^2 u}{\partial \nu^2} = 0, & \text{on } \partial\Omega, \\ \tau \frac{\partial u}{\partial \nu} - \operatorname{div}_{\partial\Omega}(D^2 u \cdot \nu) - \frac{\partial(\Delta u)}{\partial \nu} = \lambda u, & \text{on } \partial\Omega, \end{cases}$$

in the unknowns u (the eigenfunction) and λ (the eigenvalue). For $N = 2$ this problem models a free vibrating thin plate under lateral tension (the parameter τ) whose displacement at rest is described by the domain Ω and whose mass is concentrated at the boundary $\partial\Omega$. The eigenfunctions represent the natural modes of vibration of the plate while the eigenvalues are the corresponding tones. We prove that among all bounded domain of class C^1 with a fixed volume, the ball maximizes the fundamental tone (the first positive eigenvalue). The same result has been proved in [2, Chasman] for a Neumann-type problem for the biharmonic operator which models a free vibrating plate whose mass is displaced on the whole of Ω . We highlight the strict relation between these two problems by showing convergence of the spectrum of a class of Neumann-type problems for the biharmonic operator to the spectrum of problem (1).

BIBLIOGRAPHY

- [1] C. Bandle, *Isoperimetric inequalities and applications*. Pitman advanced publishing program, monographs and studies in mathematics, vol. 7, 1980.
- [2] L.M. Chasman, *An isoperimetric inequality for fundamental tones of free plates*. Comm. Math. Phys. 303 (2011), no. 2, 421449.
- [3] D. Buoso, L. Provenzano, *An isoperimetric inequality for the first non-zero eigenvalue of the biharmonic Steklov problem*, in preparation.
- [4] P.D. Lamberti, L. Provenzano, *Viewing the Steklov eigenvalues of the Laplace operator as critical Neumann eigenvalues*, submitted to the 9th Isaac Congress proceedings.