

Luigi Provenzano Institut de Mathématiques, Université de Neuchâtel, Rue Emile-Argand 11, 2000 Neuchâtel, Switzerland, email: luigi.provenzano@unine.ch

Eigenvalues of the biharmonic operator with density

Abstract

We consider the eigenvalues of the biharmonic operator subject to Neumann boundary conditions and with a positive density ρ on a domain Ω of the Euclidean space \mathbb{R}^N , namely problem

$$\begin{cases} \Delta^2 u = \lambda \rho u, & \text{in } \Omega, \\ \frac{\partial^2 u}{\partial \nu^2} = \operatorname{div}_{\partial\Omega} (D^2 u \cdot \nu) + \frac{\partial \Delta u}{\partial \nu} = 0, & \text{on } \partial\Omega. \end{cases}$$

We study the dependence of the eigenvalues upon the density ρ and the domain Ω under the constraint $\int_{\Omega} \rho dx = |\Omega|$. We prove uniform upper bounds which respect the Weyl's law for all the Neumann eigenvalues in dimension $N \geq 4$ by exploiting a technique developed in [2, 3] and recently used to prove upper bounds for the eigenvalues of the Laplacian on manifolds (see e.g., [1]). Finally, we shall provide some remarks on the case $N = 2, 3$, on Dirichlet or intermediate conditions, on extensions to polyharmonic operators and on extension to manifolds. Based on a joint work with Prof. Bruno Colbois.

BIBLIOGRAPHY

- [1] B. Colbois; A. El Soufi; A. Savo. *Eigenvalues of the Laplacian on a compact manifold with density*. Comm. Anal. Geom. 23 (2015), no. 3, 639–670.
- [2] A. Grigor'yan; Y. Netrusov; S-T. Yau. *Eigenvalues of elliptic operators and geometric applications*. Surveys in differential geometry. Vol. IX, 147–217, Surv. Differ. Geom., IX, Int. Press, Somerville, MA, 2004.
- [3] N. Korevaar. *Upper bounds for eigenvalues of conformal metrics*. J. Differential Geom. 37 (1993), no. 1, 73–93.